King Abdulaziz University
Faculty of Science
Department of Mathematics

قسم الرياضيات
Department of Mathematics
King Abdulaziz University

Math 206-Exam. I
Third Semester (2022-23)
April 10, 2023

Time Allowed: 90 minutes

B

Max. Marks: **60**

Name:____ **ID. No.**________________________

Instructor's Name: ___________________________________ **Section:** ______________**Class Time** __________

Question	1	2	3	4	5	**Total marks (out of 60)**
Marks Obtained						

Marks out of 30

Instructions: 1) Attempt all FIVE (5) questions.

2) Calculators and electronic devices are not allowed.

3) Switch of your mobile(s) and put in your pocket during examination period.

Q.1 **(a)** If $y = \cosh(\cos x) + \tanh^{-1}(\sqrt{x})$, then find $\frac{dy}{dx}$. (5 marks)

$$y' = -\sinh(\cos x)\cdot\sin x + \frac{\frac{1}{2\sqrt{x}}}{1-x} \qquad \frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$y' = -\sinh(\cos x)\cdot\sin x + \frac{1}{(1-x)2\sqrt{x}}$$

$$y' = -\sinh(\cos x)\cdot\sin x + \frac{1}{2\sqrt{x} - 2x\sqrt{x}}$$

(b) Find the limit $\lim_{x\to\infty}\left(x^3 e^{-x^3}\right)$ $\infty\cdot\frac{1}{e^{\infty}} : \infty\cdot 0$ I.D.F (5 marks)

$$\lim_{x\to\infty}\frac{x^3}{e^{x^3}} = \frac{\infty}{\infty} \quad \text{I.D.F}$$

LH

$$\lim_{x\to\infty}\frac{\cancel{3x^2}}{\cancel{3x^2}e^{x^3}}$$

$$\lim_{x\to\infty}\frac{1}{e^{x^3}} = \frac{1}{\infty} = 0$$

Q.2 **(a)** Express the integral $\int_{0}^{\frac{\pi}{4}} (3 + \tan x)\, dx$ as a **limit of sums.** (5 marks)

b, a

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$\Delta x = \frac{b-a}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} 3 + \tan \frac{\pi}{4n} i \quad \frac{\pi}{4n}$$

$$\Delta x = \frac{\frac{\pi}{4}}{n} = \frac{\pi}{4n}$$

$$x_i = a + \Delta x\, i$$

$$\lim_{n\to\infty} \frac{\pi}{4n} \sum_{i=1}^{n} 3 + \tan \frac{\pi}{4n} i$$

$$x_i = \frac{\pi}{4n} i$$

$$f(x_i) = 3 + \tan \frac{\pi}{4n} i$$

(b) Evaluate $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$ (5 marks)

$$\sqrt{x} \rightarrow \frac{1}{2\sqrt{x}}$$

$$2 \int e^{\overset{f(x)}{\sqrt{x}}} \cdot \underbrace{\frac{1}{2\sqrt{x}}}_{f'(x)}\, dx$$

$$= 2e^{\sqrt{x}} + C$$

LH

L.H

Q.3 **(a)** Evaluate $\int \frac{\sec^2 x}{\sqrt{3+\tan x}}\, dx$ (5 marks)

$$\int \underbrace{\sec^2 x}_{f'(x)} \; \underbrace{(3+\tan x)}_{f(x)}^{-\frac{1}{2}} \; dx$$

$$= \frac{(3+\tan x)^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= 2(3+\tan x)^{\frac{1}{2}} + C$$

$$= 2\sqrt{3+\tan x} + C$$

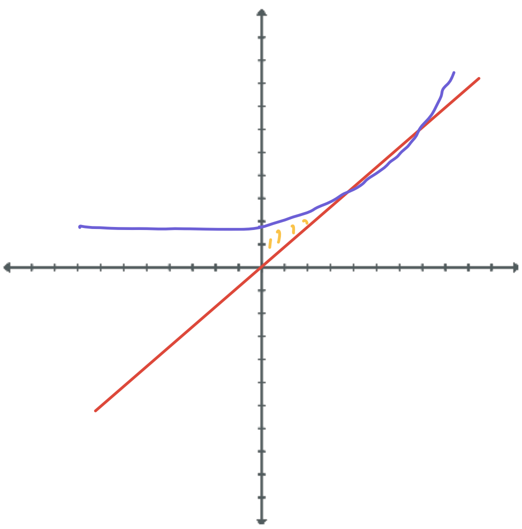
(b) Find the **area** of the bounded region $y = x$, $y = e^x$, $0 \leq x \leq 2.$ (5 marks)

$$A = \int_0^2 e^x - x \quad dx$$

$$A = \left[e^x - \frac{x^2}{2} \right]_0^2$$

$$A = e^2 - 2 - 1$$

$$A = e^2 - 3$$



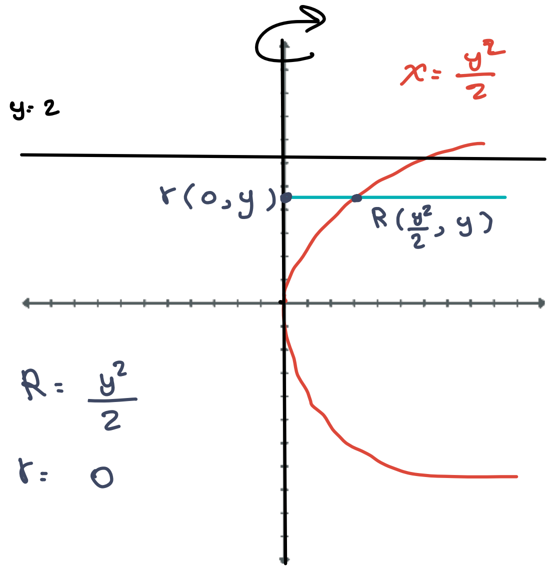
$y = x$, $y = e^x$

$y = 1$, $y = e$

$y = x < y = e^x$

Q.4 **(a)** Find the **volume** of the solid obtained by rotating the bounded region $2x = y^2$, $x = 0$, $0 \le y \le 2$; about the ***y*-axis.** (5 marks)

$x = \frac{y^2}{2}$



$V = \int_0^2 \pi [R^2 - r^2] \, dy$

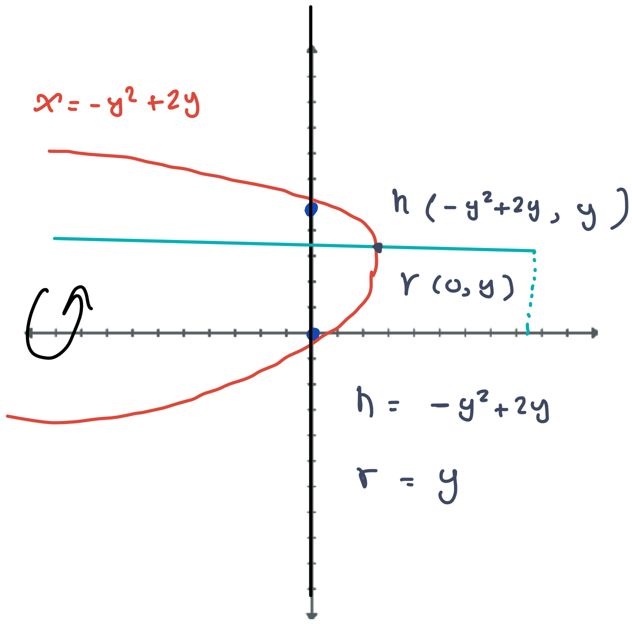
$V = \pi \int_0^2 \frac{y^4}{4} \, dy$

$V = \pi \left[\frac{y^5}{20} \right]_0^2$

$V = \frac{32\pi}{20} = \frac{16\pi}{10} = \frac{8\pi}{5}$

(b) Find the **volume** of the solid obtained by rotating the bounded region $x = -y^2 + 2y$, $x = 0$, $0 \le y \le 2$; about the ***x*-axis.** (5 marks)

(Hint: Use shell method)



$V = \int_0^2 2\pi r h \, dy$

$V = 2\pi \int_0^2 (-y^2+2y)(y) \, dy$

$V = 2\pi \int_0^2 -y^3 + 2y^2 \, dy$

$V = 2\pi \left[-\frac{y^4}{4} + \frac{2y^3}{3} \right]_0^2$

$V = 2\pi \left[-4 + \frac{16}{3} \right]$

$V = 2\pi \left[\frac{4}{3} \right] = \frac{8\pi}{3}$

$x = -y^2 + 2y$

$x = 0$ $y = 0$, $y = 2$

Q.5 Choose the **correct** answer for each of the following statements. (20 marks)

i) If $\int_1^4 f(x)\,dx = 3$ and $\int_1^4 g(x)\,dx = \boxed{2}$,
then $\int_1^4 [3f(x) - 2g(x)]\,dx = 5$. $\boxed{\text{a) True}}$ b) False

$\underbrace{\cdot 3 \qquad \cdot 2}_{9\ -\ 4}$

ii) $\sinh^2 x = 1 + \cosh^2 x$ $\cosh^2 - \sinh^2 = 1$ a) True $\boxed{\text{b) False}}$

$\cosh^2 - \sinh^2 = -1$

iii) $\int_1^e \frac{1}{x}\,dx = 1$ $[\ln|x|]_1^e$ $\boxed{\text{a) True}}$ b) False

$1 - 0 = 1$

iv) $\frac{d}{dx}\left[\int_{\cancel{\ln x}\ x^2}^{1} e^t\,dt\right] = -x$ a) True $\boxed{\text{b) False}}$

$\frac{\cos x}{\sin}$

v) $\int \cos x \csc x\,dx = \frac{\cos^2 x}{2} + C$ a) True $\boxed{\text{b) False}}$

$\int \cos x\,(\sin x)^{-1} = \ln|\sin x|$

vi) $\lim_{x\to\infty}\left(\frac{1}{x} - \frac{1}{x^2}\right) = 1$ a) True $\boxed{\text{b) False}}$

$0 - 0 = 0$

vii) $\lim_{x\to\infty}\left(\frac{\ln x}{x}\right) = 0$ $\frac{\infty}{\infty} = \frac{\frac{1}{x}}{1} = \frac{1}{x} = 0$ $\boxed{\text{a) True}}$ b) False

$\frac{f'}{f}$

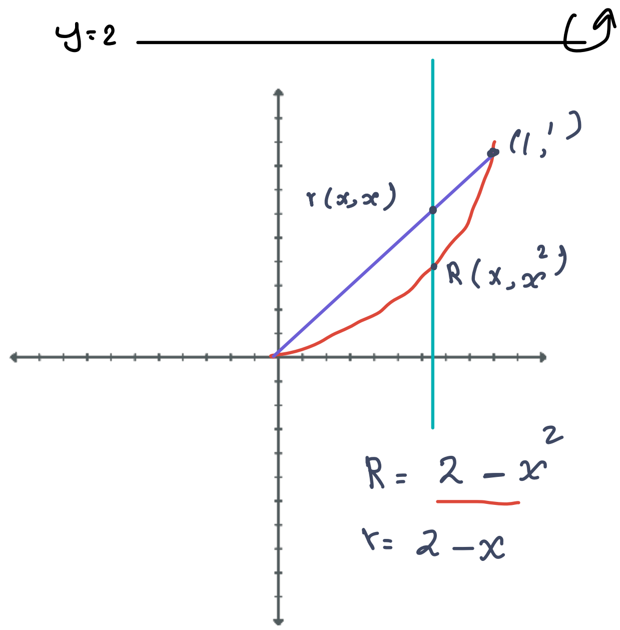
viii) $\int \tan x\,dx = \ln|\sin x| + C$ a) True $\boxed{\text{b) False}}$

ix) $\int 2x\,e^{(x^2+1)}\,dx = \frac{1}{2}e^{(x^2+1)} + C$ a) True $\boxed{\text{b) False}}$

x) The integral that represents the volume of the solid obtained by rotating the region bounded by $y = x^2$, $x = y$, $0 \le x \le 1$; about the line $y = 2$ is;
$V = \int_0^1 \pi\,[(2 - x^2)^2 - (2 - x)^2]\,dx$. $\boxed{\text{a) True}}$ b) False

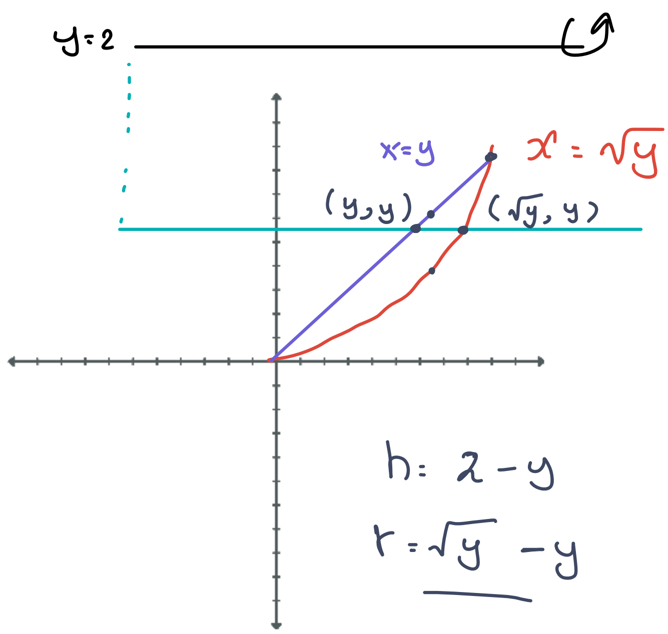
$$A = \pi \int_0^1 [R^2 - r^2]\, dx$$

$$\pi \int_0^1 (2-x^2)^2 - (2-x)^2 \, dx$$



$$A = 2\pi \int_0^1 r h \quad dy$$

$$A = 2\pi \int_0^1 (\sqrt{y} - y)(2-y)\, dy$$



$$y = \sqrt{y}$$

$$y^2 = y$$

$$y^2 - y = 0$$

$$y(y-1) = 0$$

$$\int \frac{x}{\sqrt{x+2}}\,dx$$

$u = x+2$

$du = 1\,dx$

$x = u-2$

$$\int \frac{u-2}{\sqrt{u}}\,du$$

$$\int \frac{u}{\sqrt{u}}\,du - 2\int \frac{1}{\sqrt{u}}\,du$$

$$\int u^{\frac{1}{2}}\,du - 2\int u^{-\frac{1}{2}}\,du$$

$$\frac{2}{3}u^{3/2} - 4u^{1/2} + C$$

$$\frac{2}{3}(x+2)^{3/2} - 4(x+2)^{\frac{1}{2}} + C$$

$$\int \frac{1}{4+x^2}\,dx = \frac{1}{2}\tan^{-1}\frac{x}{2} + C$$

$$\int \frac{3}{5+x^2}\,dx = 3\int \frac{1}{5+x^2}\,dx$$

$$= 3\cdot\frac{1}{\sqrt{5}}\tan^{-1}\frac{x}{\sqrt{5}} + C.$$

$$\int \frac{9}{9+x^2}\,dx$$

$$9\int \frac{1}{9+x^2}\,dx$$

$$= 9\cdot\frac{1}{3}\tan^{-1}\frac{x}{3} + C.$$