# CH 12.2

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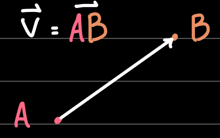
## Vectors

A vector is a quantity that has both a magnitude and direction; represented by an arrow; denoted by a boldface letter "**v**" or an arrow over a letter "$\vec{v}$"; has initial point "A" (the tail) and terminal point "B" (the tip).

$\vec{v} = \overrightarrow{AB}$



# Components of a vector

Each vector has components that are denoted as follows,

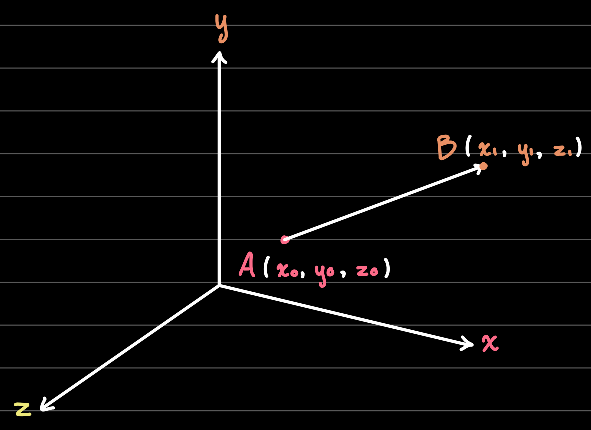
$$\langle a, b \rangle \text{, in } \mathbb{R}^2$$

$$\langle a, b, c \rangle \text{, in } \mathbb{R}^3$$

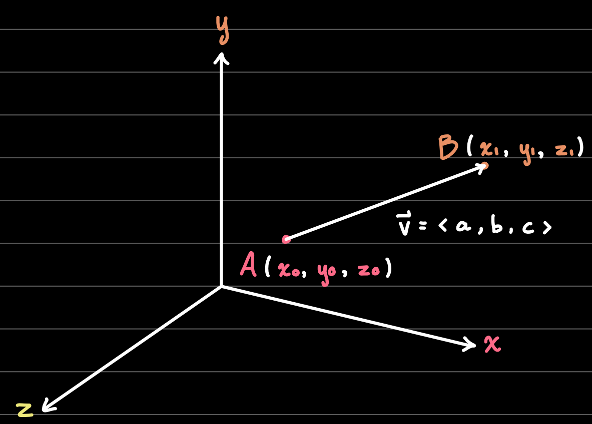
Don't confuse it with the notation of a point "(a, b, c)".

Now, say there were points "$A(x_0, y_0, z_0)$" and "$B(x_1, y_1, z_1)$" where

$$x_1 = a + x_0 \quad , \quad y_1 = b + y_0 \quad , \quad z_1 = c + z_0$$




As mentioned previosly, a vector has a terminal point and an initial point. In our case, let "A" be the initial point, and "B" be the terminal.

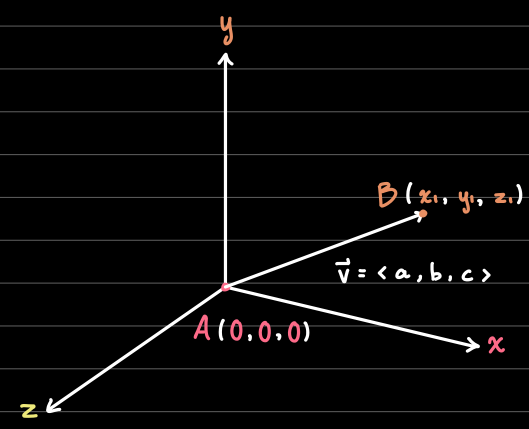


To determine the components of "$\vec{v}$",

$$a = x_1 - x_0 \quad , \quad b = y_1 - y_0 \quad , \quad c = z_1 - z_0$$

$$\vec{v} = \langle x_1 - x_0 , y_1 - y_0 , z_1 - z_0 \rangle$$

By the same logic, if a vector's tail is the origin point then,




$$a = x_1 - 0 \quad , \quad b = y_1 - 0 \quad , \quad c = z_1 - 0$$

$$a = x_1 \quad , \quad b = y_1 \quad , \quad c = z_1$$

$$\vec{v} = \langle x_1 , y_1 , z_1 \rangle$$

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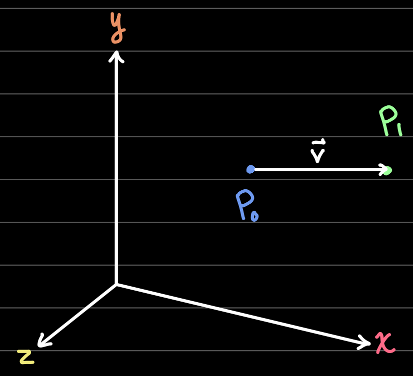
# Magnitude of a vector

Let there be a vector "$\vec{v}$" with tail "$P_0$" and tip "$P_1$", where,

$\vec{v} = \langle a, b, c \rangle$ , $P_0 = (x_0, y_0, z_0)$ , $P_1 = (x_1, y_1, z_1)$




To compute the length of "$\vec{v}$", we need to figure out the distance from "$P_0$" to "$P_1$".

$$|P_0P_1| = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$$

Recall that,

$$a = x_1 - x_0 \quad , \quad b = y_1 - y_0 \quad , \quad c = z_1 - z_0$$

$$\vec{v} = \langle x_1 - x_0 , y_1 - y_0 , z_1 - z_0 \rangle$$

$$= \langle a, b, c \rangle$$

Therefore,

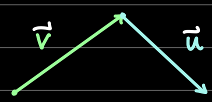
Note: the length of a vector is denoted by either $|\vec{v}|$ or $\|\vec{v}\|$.

$$|\vec{v}| = \sqrt{(a)^2 + (b)^2 + (c)^2}$$

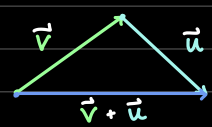
# Summing vectors

Let there be two vectors "$\vec{v}$" and "$\vec{u}$" where "$\vec{v}$" tip is at the tail of "$\vec{u}$".

$$\vec{v} = \langle a, b, c \rangle \quad , \quad \vec{u} = \langle d, e, f \rangle$$




The sum of "$\vec{v}$" and "$\vec{u}$" would result in a vector that begins from the tail of "$\vec{v}$" and ends at the tip of "$\vec{u}$".




$$\vec{v} + \vec{u} = \langle a + d \, , \, b + e \, , \, c + f \rangle$$

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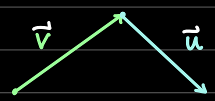
# Subtracting vectors

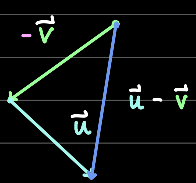
Let there be two vectors "$\vec{v}$" and "$\vec{u}$" where "$\vec{v}$" tip is at the tail of "$\vec{u}$".

$$\vec{v} = \langle a, b, c \rangle \quad , \quad \vec{u} = \langle d, e, f \rangle$$




Then vector subtracting vector "$\vec{v}$" from "$\vec{u}$" would be the same as summing "$-\vec{v}$" and "$\vec{u}$".

$$\vec{u} - \vec{v} = \vec{u} + (-\vec{v})$$

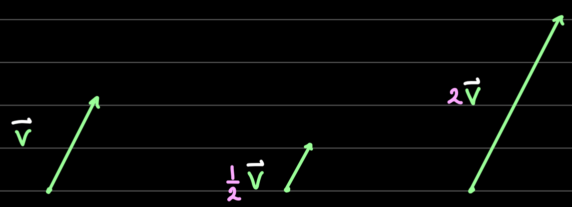


# Multiplying vectors by scalar

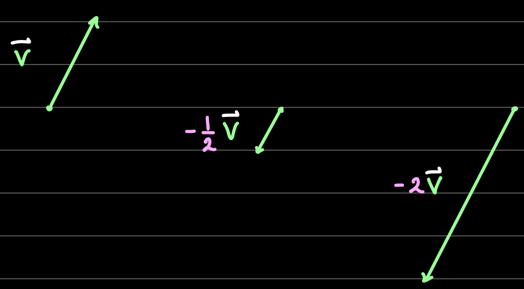
Let "$k$" be a scalar, and "$\vec{v}$" a vector. Then " $\vec{v}$ " is called a scalar multiplication.

$$k\vec{v} = k<a, b, c> = <ka, kb, kc>$$

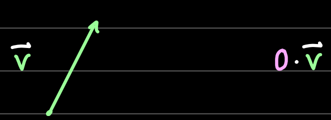
If "$k>0$", then




If "$k<0$", then




If "$k=0$", then



# Properties of vectors

1) $\vec{v} + \vec{u} = \vec{u} + \vec{v}$

2) $(\vec{v} + \vec{u}) + \vec{w} = \vec{v} + (\vec{u} + \vec{w})$

3) $\vec{v} + 0 = \vec{v}$

4) $\vec{v} + (-\vec{v}) = 0$

5) $k \cdot (\vec{v} + \vec{u}) = k\vec{v} + k\vec{u}$

6) $\vec{v} \cdot (k + g) = k\vec{v} + g\vec{v}$

7) $(k \cdot g) \cdot \vec{v} = k \cdot (\vec{v} \cdot g)$

8) $1\vec{v} = \vec{v}$

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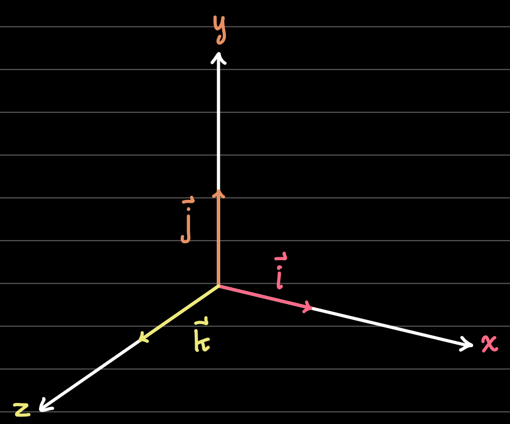
# Standard basis vectors

The vectors "$\vec{i}$", "$\vec{j}$", and "$\vec{k}$" are called the standard basis vectors and point in the directions of the positive x, y, and z-axis

$\vec{i} = \langle 1, 0, 0 \rangle$ , $\vec{j} = \langle 0, 1, 0 \rangle$ , $\vec{k} = \langle 0, 0, 1 \rangle$




These standard vectors allow us to rewrite vectors

$$\vec{v} = \langle a, b, c \rangle = a_i + b_j + c_k$$

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# Unit vectors

A unit vector is a vector with a length of 1.

Any vector becomes a unit vector if it is divided by its own length.

$$\vec{u} = \frac{1}{\|\vec{v}\|} \cdot \vec{v}$$