✨ CH 12.1 ✨

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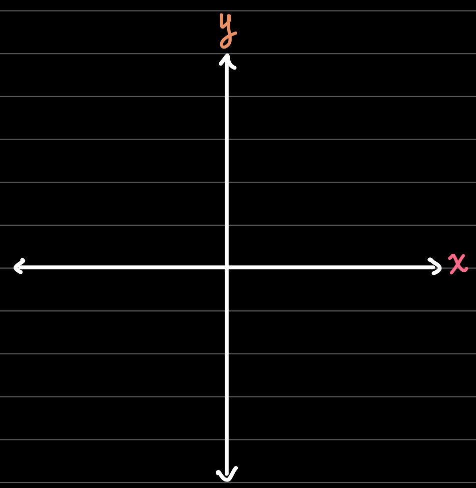
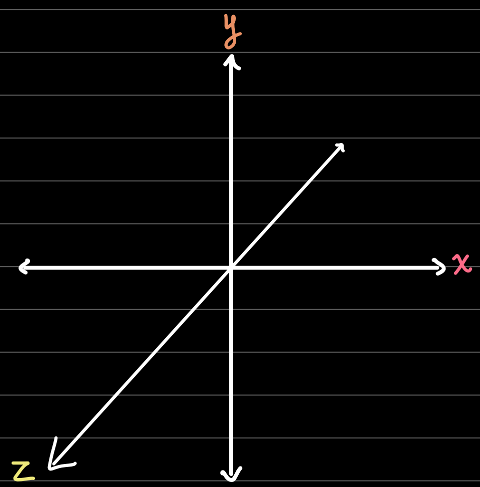
Three-dimensional coordinate systems

In a two-dimensional coordinate system, a point is represented by an ordered pair of real numbers (a, b), where "a" is associated with the x-axis, and "b" is associated with the y-axis.



In a three-dimensional coordinate system, we add a third real number (a, b, c). "c" is associated with the z-axis.



This type of represention of points is called the Cartesian product:

For two-dimensions:

$$\mathbb{R} \times \mathbb{R} : \{ (x, y) \in \mathbb{R} \}, \text{ denoted by } \text{"}\mathbb{R}^2\text{"}.$$

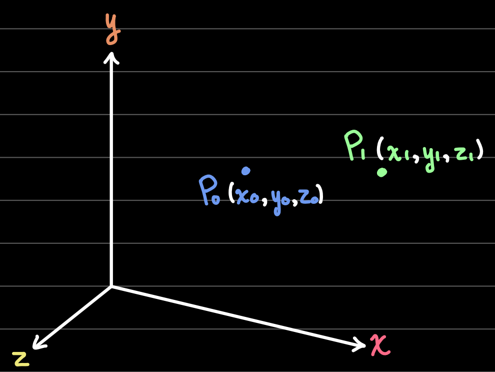
For three-dimensions:

$$\mathbb{R} \times \mathbb{R} \times \mathbb{R} : \{ (x, y, z) \in \mathbb{R} \}, \text{ denoted by } \text{"}\mathbb{R}^3\text{"}.$$

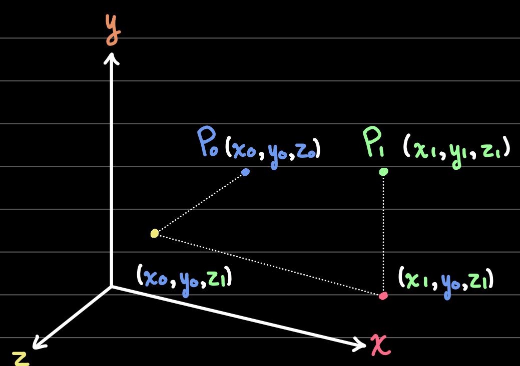
Note: the graph of an equation in "$\mathbb{R}^2$" is called a curve. In "$\mathbb{R}^3$", it is called a surface.

Distance

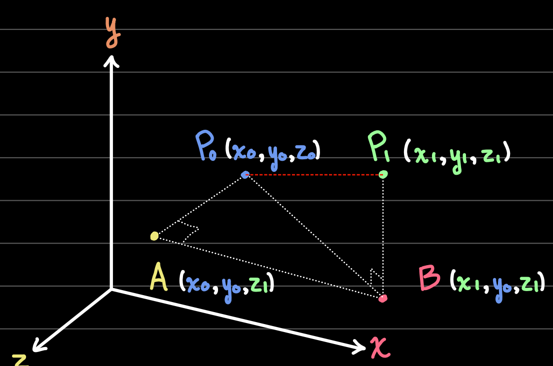
Let there be points "P_0" and "P_1", where "P_0" is at "(x_0, y_0, z_0)", and "P_1" is at "(x_1, y_1, z_1)".



We can make it from point "P_0" to "P_1" by moving in parallel directions to each axis.

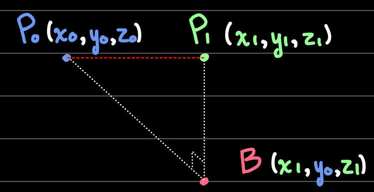


By observing, it is apperant that we can draw right angled triangles. Also, let's name point "(1,2,6)" "A", and point "(4,2,6)" "B".



$|P_0A| = |z_1 - z_0|$ $\qquad$ $|AB| = |x_1 - x_0|$ $\qquad$ $|BP_1| = |y_1 - y_0|$

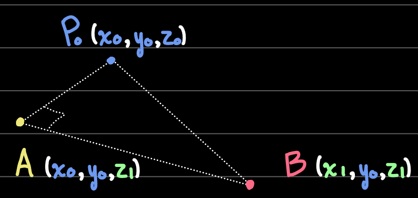
The distance from "P_0" to "P_1" would the hypotenuse of the triangle $\angle P_0BP_1$



$$|P_0P_1|^2 = |P_0B|^2 + |BP_1|^2$$

The distance from "P_1" to "B" would the hypotenuse of the triangle $\angle P_1AB$

$$|P_0B|^2 = |P_0A|^2 + |AB|^2$$



Replacing "$|P_0B|^2$" with its value in the equation of the distance from "P_0" to "P_1".

$$|P_0P_1|^2 = |P_0B|^2 + |BP_1|^2$$

$$= |P_0A|^2 + |AB|^2 + |BP_1|^2$$

Then we square root both sides,

$$|P_0P_1| = \sqrt{|P_0A|^2 + |AB|^2 + |BP_1|^2}$$

After replacing the variables with their values,

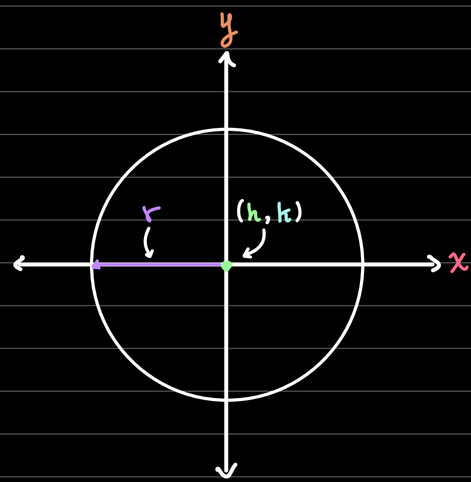
$$= \sqrt{|z_1 - z_0|^2 + |x_1 - x_0|^2 + |y_1 - y_0|^2}$$

Since they are being squared, there is no need for the absaloute value,

$$|P_0 P_1| = \sqrt{(z_1 - z_0)^2 + (x_1 - x_0)^2 + (y_1 - y_0)^2}$$

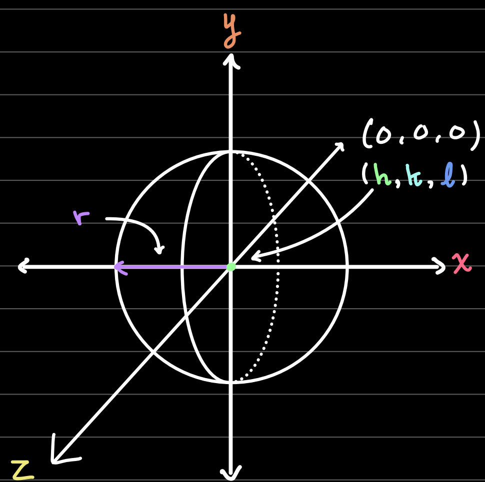
Spheres

Recall the equation of a circle.



$$(x-h)^2 + (y-k)^2 = r^2$$

Now, all we have to do to turn a circle into a sphere is add the "z" component into the equation



$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$$