

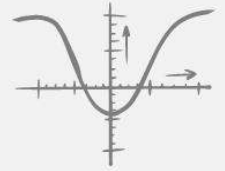
**BRAVO**

goals

$\int x$

$\frac{x}{y}$

$\div$



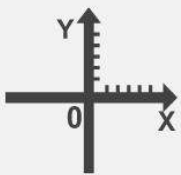
# مقرر الرياضيات MATH-110

## Appendix D

$\pi$

Trigonometry

$\sqrt{x}$



$fx$

$|x|$

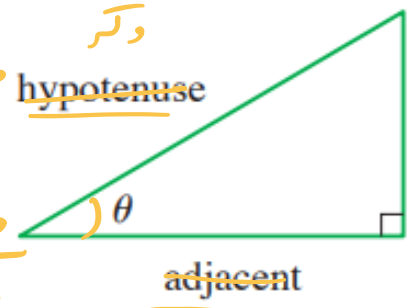
$X_n$

ابدأ التعلم الآن

# Trigonometric functions (Appendix D)

(الدوال المثلثية)

## 5- Trigonometric function



$\sin \theta = \frac{\text{opp}}{\text{hyp}}$      $\csc \theta = \frac{\text{hyp}}{\text{opp}}$   
 $\cos \theta = \frac{\text{adj}}{\text{hyp}}$      $\sec \theta = \frac{\text{hyp}}{\text{adj}}$   
 $\tan \theta = \frac{\text{opp}}{\text{adj}}$      $\cot \theta = \frac{\text{adj}}{\text{opp}}$

hypotenuse (وتر)    opposite (مقابل)  
 adjacent (جوار)    adjacent (جوار)  
 opposite (مقابل)    opposite (مقابل)

## Pythagorean theorem

$$r^2 = x^2 + y^2 \quad \rightarrow \quad r = \sqrt{x^2 + y^2}$$

$$x = \pm \sqrt{r^2 - y^2} \quad \quad y = \pm \sqrt{r^2 - x^2}$$

## Trigonometric identities

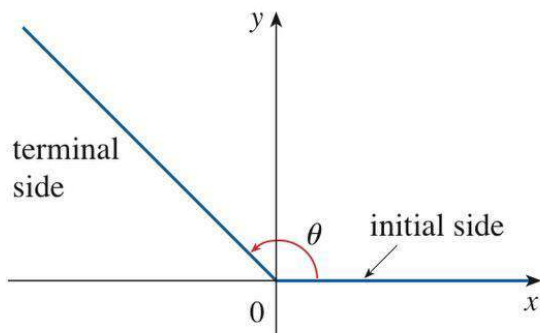
$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

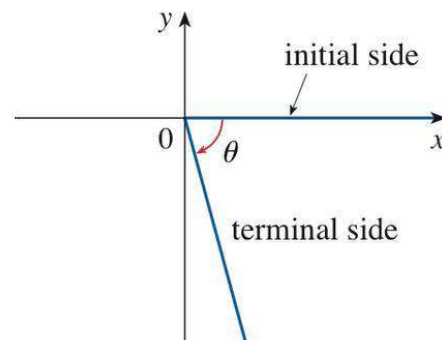
# Trigonometric functions (Continue)

(الدوال المثلثية)

## Angles can be

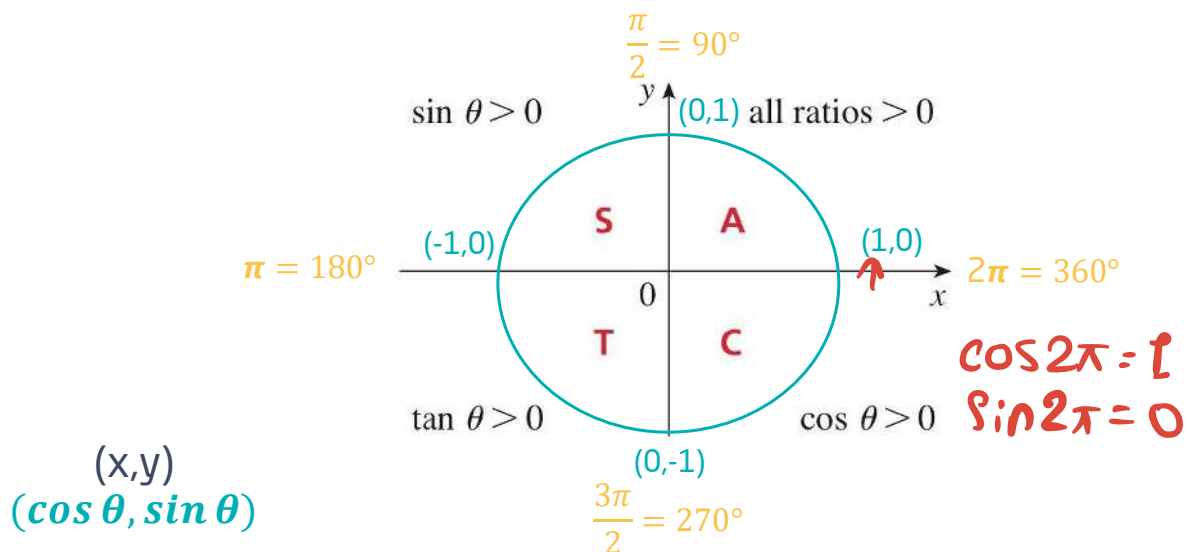


Positive  $\theta \geq 0$



Negative  $\theta < 0$

## Circle unit



# Trigonometric functions (continue)

(الدوال المثلثية)

## Trigonometric identities

$$\sin^2\theta + \cos^2\theta = 1$$

$\div \cos^2\theta$

$$\tan^2\theta + 1 = \sec^2\theta$$

$\div \sin^2\theta$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2x - \sin^2x$$

$$\sin(\theta \pm 2\pi) = \sin\theta$$

$$\cos(\theta \pm 2\pi) = \cos\theta$$

$$\cos 2x = 2 \cos^2x - 1$$

$$\cos 2x = 1 - 2 \sin^2x$$

$$\tan(\theta \pm \pi) = \tan\theta$$

$$\cos^2x = \frac{1 + \cos 2x}{2} = \frac{1}{2}(1 + \cos 2x)$$

$$\sin^2x = \frac{1 - \cos 2x}{2} = \frac{1}{2}(1 - \cos 2x)$$

# Trigonometric functions (continue)

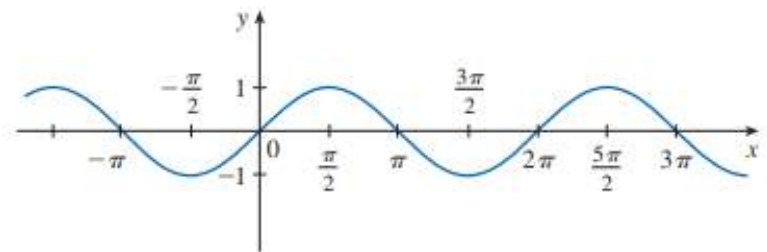


(الدوال المثلثية)

1.  $f(x) = \sin \theta$

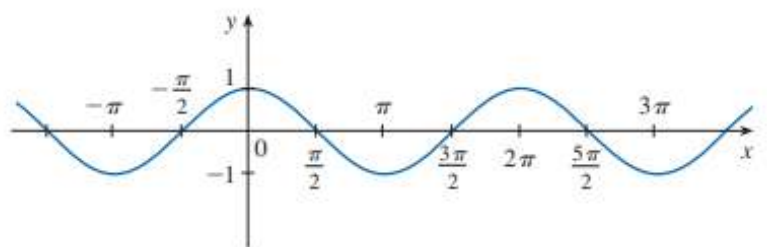
$\sin \frac{\pi}{2}$  not trig func.  
 $\Rightarrow$  constant  $\frac{1}{2}$

- Domain is  $\mathbb{R} = (-\infty, \infty)$
- Range =  $[-1, 1]$
- Period  $2\pi$
- Odd function



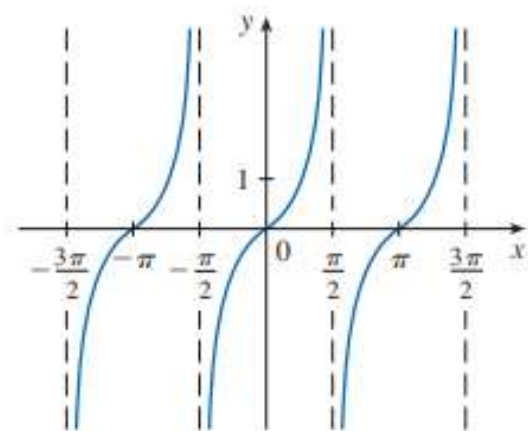
2.  $f(x) = \cos \theta$

- Domain is  $\mathbb{R} = (-\infty, \infty)$
- Range =  $[-1, 1]$
- Period  $2\pi$
- Even function



3.  $f(x) = \tan \theta$

- Domain =  $\mathbb{R} - \{\cos \theta = 0\}$   
 $= \mathbb{R} - \left\{ \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2} \dots \right\}$
- Range =  $(-\infty, \infty)$
- Period  $\pi$
- odd function



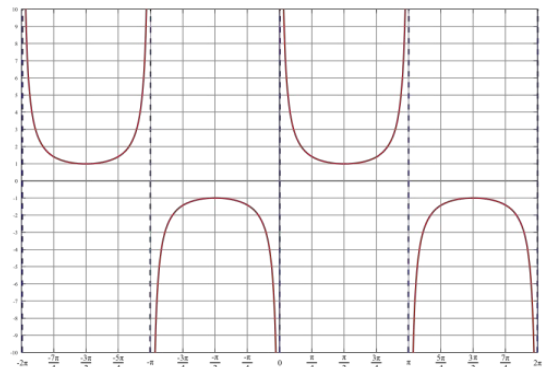
# Trigonometric functions (continue)



(الدوال المثلثية)

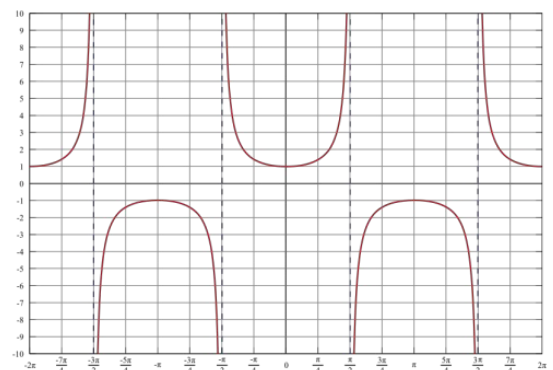
## 4. $f(x) = \csc \theta$

- Domain  $\mathbb{R} - \{0, \pm\pi, \pm2\pi, \dots\}$
- Range =  $\mathbb{R} - (-1, 1)$
- Period  $2\pi$
- Odd function



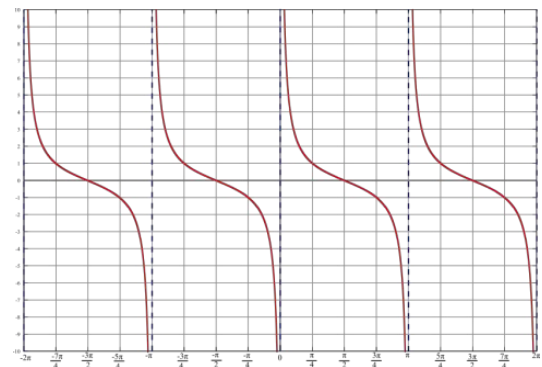
## 5. $f(x) = \sec \theta$

- Domain =  $\mathbb{R} - \left\{ \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2} \dots \right\}$
- Range =  $\mathbb{R} - (-1, 1)$
- Period  $2\pi$
- Even function



## 6. $f(x) = \cot \theta$

- Domain  $\mathbb{R} - \{0, \pm\pi, \pm2\pi, \dots\}$
- Range =  $(-\infty, \infty)$
- Period  $\pi$
- Odd function



# Trigonometric functions (continue)

(الدوال المثلثية)



## Degree ↔ Rad

- Radian → Degrees

$$\theta * \frac{180}{\pi}$$

Ex.  $\frac{\pi}{2} = 90^\circ$

- Degrees → Radian

$$\theta * \frac{\pi}{180}$$

Ex.  $30^\circ = \frac{\pi}{6}$

*Handwritten note:  $30 \times \frac{\pi}{180} = \frac{\pi}{6}$*

### Example 5

Find the domain of the function  $f(x) = \frac{1}{1 - 2 \cos x}$

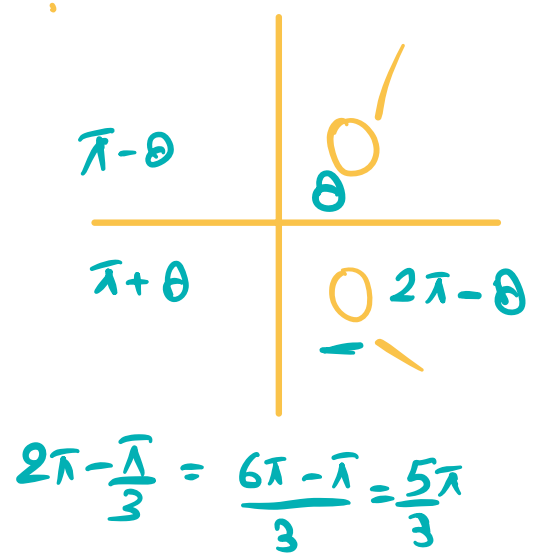
*Handwritten work:*

$$1 - 2 \cos x \neq 0$$

$$1 \neq 2 \cos x$$

$$\cos x \neq \frac{1}{2}$$

$$x \neq \frac{\pi}{3} + 2n\pi$$



*Handwritten domain:*

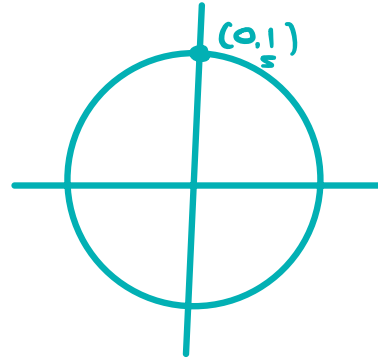
$$D = \{x \mid x \neq \frac{\pi}{3} + 2n\pi, x \neq \frac{5\pi}{3} + 2n\pi\}$$



# Trigonometric functions (continue)

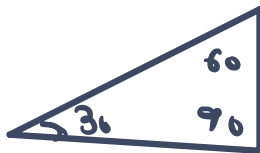
(الدوال المثلثية)

## Important values



$\sin 45 = \frac{1}{\sqrt{2}}$

$\theta$	$0^\circ$	$30^\circ = \frac{\pi}{6}$	$45^\circ = \frac{\pi}{4}$	$60^\circ = \frac{\pi}{3}$	$90^\circ$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$ <small><math>\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}</math></small>	1	$\sqrt{3}$	Not defined
$\cot \theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\operatorname{cosec} \theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1



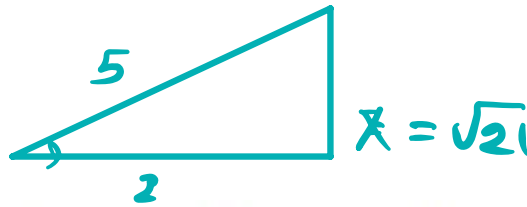


# Trigonometric functions (continue)

(الدوال المثلثية)

App. D

## Example 4



★ If  $\cos \theta = \frac{2}{5}$  and  $0 < \theta < \pi/2$ , find the other five trigonometric functions of  $\theta$ .

$$\begin{aligned} 5^2 &= 2^2 + x^2 \\ 25 &= 4 + x^2 \\ x^2 &= 21 \\ \Rightarrow x &= \sqrt{21} \end{aligned}$$

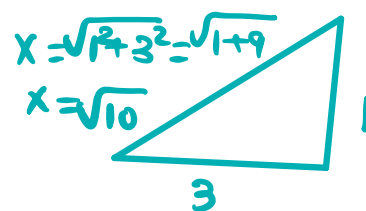
$$\begin{aligned} \sin \theta &= \frac{\sqrt{21}}{5} \\ \tan \theta &= \frac{\sqrt{21}}{2} \\ \csc \theta &= \frac{5}{\sqrt{21}} \end{aligned}$$

$$\begin{aligned} \sec \theta &= \frac{5}{2} \\ \cot \theta &= \frac{2}{\sqrt{21}} \end{aligned}$$

## Exercise 33

Find the remaining trigonometric ratios.

$$\cot \beta = \frac{3}{1}, \quad \pi < \beta < \frac{3\pi}{2}$$



$$\sin \beta = -\frac{1}{\sqrt{10}}$$

$$\cos \beta = -\frac{3}{\sqrt{10}}$$

$$\tan \beta = \frac{1}{3}$$

$$\csc \beta = -\sqrt{10}$$

$$\sec \beta = \frac{-\sqrt{10}}{3}$$

$$\begin{array}{c|c} \text{S} & \text{C} \\ \hline + & - \\ \uparrow & \uparrow \end{array}$$



تأكد دائمًا ان  
#الدافور\_معك!

**The End**

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