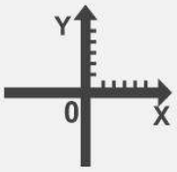
BRAVO Me

goals

مقرر الرياضيات
MATH-110

Appendix B

Coordinates geometry and lines



ابدأ التعلم الآن

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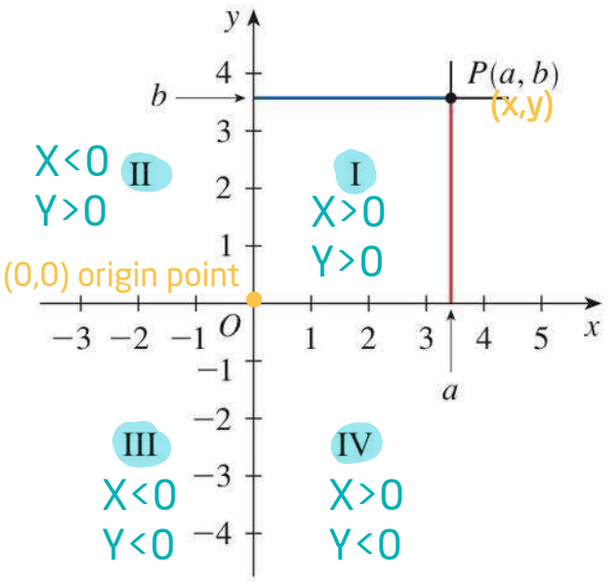
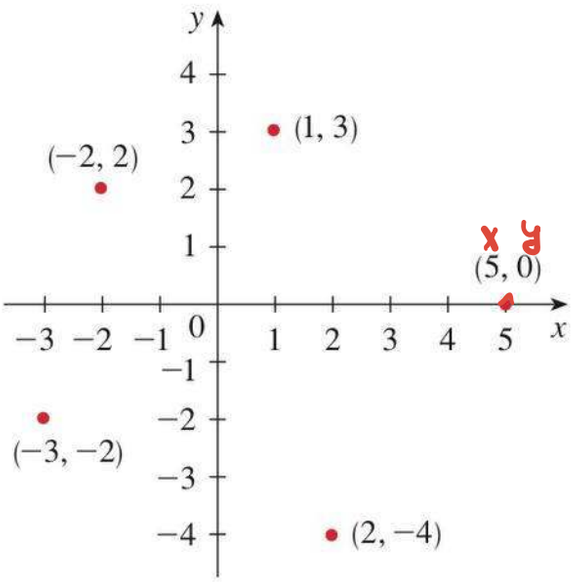
Coordinate Geometry

XY plane or Cartesian plane:

Y-axis: x=0



X-axis: y=0

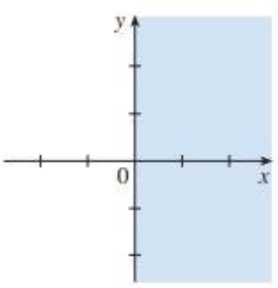


- (XY plane) or (Cartesian plane) or (Coordinate plane) is denoted by $\mathbb{R}^2$

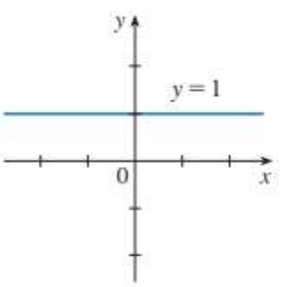
EXAMPLE 1:

Sketch the regions given by the following sets.

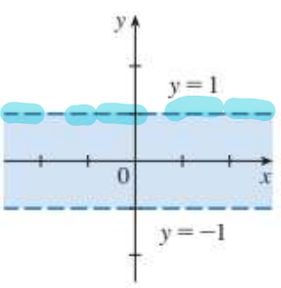
(a) $\{(x, y) \mid x \geq 0\}$ (b) $\{(x, y) \mid y = 1\}$ (c) $\{(x, y) \mid |y| < 1\}$



(a) $x \geq 0$



(b) $y = 1$



(c) $|y| < 1$

$|y| < 1$ if and only if $-1 < y < 1$

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Coordinate Geometry and Lines (continue)

1 Distance Formula The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

EXAMPLE 2: The distance between $(1, -2)$ and $(5, 3)$ is

Solution:

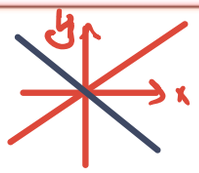
$$\sqrt{(5-1)^2 + [3-(-2)]^2} = \sqrt{4^2 + 5^2} = \sqrt{41}$$

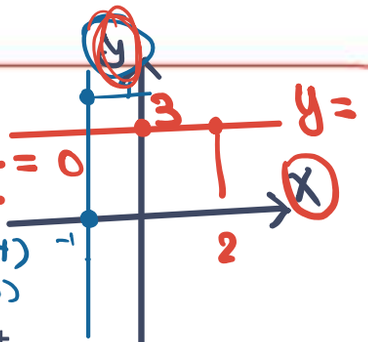
2 Definition The **slope** of a nonvertical line that passes through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of a vertical line is not defined.

NOTES:



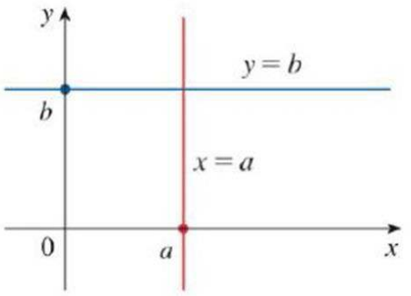


- Positive slope → upward to the right.
- Negative slope → downward to the right.
- Horizontal line → Parallel x axis → slope 0 → eq. y=b
- Vertical line → Parallel y axis → slope: Not defined → eq. x=a

Parallel slope → $m_1 = m_2$ Same

Perpendicular slope → $m_2 = -\frac{1}{m_1}$

$m_1 m_2 = -1$



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Lines (continue)

EXAMPLE: Find the slope of line through $(1, -2)$ and $(5, 3)$?

P_1 x_1 y_1 P_2 x_2 y_2

Solution:

$$\frac{y2-y1}{x2-x1} = \frac{3-(-2)}{5-1} = \frac{3+2}{4} = \frac{5}{4}$$

3 Point-Slope Form of the Equation of a Line An equation of the line passing through the point $P_1(x_1, y_1)$ and having slope m is

① ②

$$y - y_1 = m(x - x_1)$$

EXAMPLE 3: Find an equation of the line through $(1, -7)$ with slope $-\frac{1}{2}$.

P_1 x_1 y_1 m

Solution: Using (3) with $m = -\frac{1}{2}$, $x_1 = 1$, and $y_1 = -7$, we obtain an equation of the line as

$y-y_1 = m(x-x_1) \Rightarrow y-(-7) = -\frac{1}{2}(x-1)$

$$y + 7 = -\frac{1}{2}(x - 1)$$

which we can rewrite as $2y+14 = -x+1$ ①

$$2y + 14 = -x + 1 \quad \text{or} \quad x + 2y + 13 = 0$$

$x+2y+13=0$

$x+2y=-13$

EXAMPLE 4:

Find an equation of the line through the points $(-1, 2)$ and $(3, -4)$.

P_1 x_1 y_1 P_2 x_2 y_2

① m ② P.

Solution: $m = \frac{-4-2}{3+1} = \frac{-6}{4} = \frac{-3}{2}$

$y-2 = \frac{-3}{2}(x+1)$

$2y-4 = -3x-3 \Rightarrow 3x+2y=1$

$-3+4$

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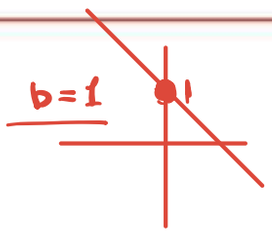
Lines (continue)

4 Slope-Intercept Form of the Equation of a Line An equation of the line with slope m and y-intercept b is

$$y = mx + b$$



EXAMPLE:

Find an equation of the line with slope 5 and y-intercept 7?

Solution: $y = mx + b \Rightarrow y = 5x + 7$

EXAMPLE 5: Sketch the graph of the equation $3x - 5y = 15$.

Solution:

Let $x = 0$

$3(0) - 5y = 15$

$-5y = 15$

$y = -3$

$(0, -3)$

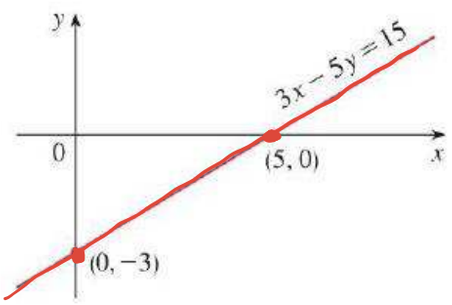
Let $y = 0$

$3x - 5(0) = 15$

$\frac{3x}{3} = \frac{15}{3}$

$x = 5$

$(5, 0)$



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Lines (continue)

EXAMPLE 6: Graph the inequality $x + 2y > 5$.

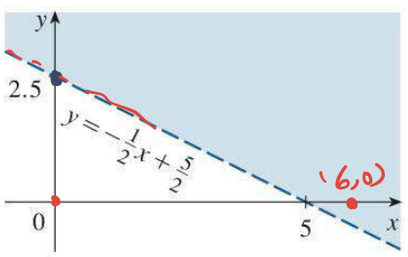
Solution:

$y = mx + b$

$\frac{2y}{2} > \frac{5 - x}{2}$

$y > \frac{-x}{2} + \frac{5}{2}$

$m = \frac{-1}{2}$ $\quad \frac{5}{2} = 2.5$



Let $x = 5$

$y = \frac{-5}{2} + \frac{5}{2} = 0$

(5,0)

$0 + 2(0) > 5$

$0 > 5$ ✗

$6 + 2(0) > 5$

$6 > 5$ ✓

EXAMPLE 7: Find an equation of the line through the point (5, 2) that is parallel to the line $4x + 6y + 5 = 0$.

x_1 y_1 Same m

Solution: $y - y_1 = m(x - x_1)$

$6y = -4x - 5$

$y = \frac{-4x}{6} - \frac{5}{6}$

$y = \frac{-2x}{3} - \frac{5}{6}$ ✗

$y - 2 = \frac{-2}{3}(x - 5)$

$3y - 6 = -2x + 10$

$3y + 2x = 16$ ✗

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Lines (continue)

EXAMPLE 8:

Show that the lines $2x + 3y = 1$ and $6x - 4y - 1 = 0$ are perpendicular.

m_1 m_2

$m_1 m_2 = -1$

Solution:

$y = mx + b$

$\frac{3y}{3} = \frac{-2x}{3} + \frac{1}{3}$

$y = \frac{-2}{3}x + \frac{1}{3}$

$m_1 = \frac{-2}{3}$

$\frac{-4y}{-4} = \frac{+6x}{-4} + \frac{1}{-4}$

$y = \frac{3}{2}x - \frac{1}{4}$

$m_2 = \frac{3}{2}$

$m_1 m_2 = \frac{-2}{3} \cdot \frac{3}{2} = -1$

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The End

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